GLOBAL PERTURBATIVE CLASSICAL SOLUTIONS OF THE BOLTZMANN EQUATION WITH LONG-RANGE INTERACTIONS

SUPERVISOR: CLÉMENT MOUHOT

The goal of the assignement is to understand a nice recent work that filled the last remaining gap in the theory of perturbative (close to equilibrirum) global classical solutions to the Boltzmann equation. The remaining open case was that of *long-range interactions*, for which the bilinear collision operator is a singular and shares some similarities with fractional diffusion operators.

The Boltzmann equation is one of the fundamental equations of physics, and at foundation of statistical mechanics. It plays a key role in the mathematical study of the entropy principle. The Cauchy problem for this equation remains poorly understood, similarly to 3D fluid equations. Global classical solutions have only been built in perturbative regimes. Apart from that, well-posedness (local-in-time) and the existence of so-called "renormalised" solutions[1] are known. In the case of long-range interactions, the singularity combined with the integral nature of the collision operator and the transport effects have resisted the attempt at constructing global perturbative solutions for several decades.

The assignement should contain (1) basics on the mathematics of the Boltzmann equation with [1], (2) the first theorem of global classical solutions for *short-range interactions* by late Ukai [2], (3) the recent theorem of Gressman-Strain [3] (see also [4]).

References

[1] Villani, C. A review of mathematical topics in collisional kinetic theory. Handbook of mathematical fluid dynamics, Vol. I, 71305, North-Holland, Amsterdam, 2002.

[2] Ukai, S. On the existence of global solutions of mixed problem for non-linear Boltzmann equation. Proc. Japan Acad. 50 (1974), 179–184.

[3] Gressman, Philip T.; Strain, Robert M. Global classical solutions of the Boltzmann equation without angular cut-off. J. Amer. Math. Soc. 24 (2011), no. 3, 771–847.

[4] Alexandre, R.; Morimoto, Y.; Ukai, S.; Xu, C.-J.; Yang, T. Global well-posedness theory for the spatially inhomogeneous Boltzmann equation without angular cutoff. C. R. Math. Acad. Sci. Paris 348 (2010), no. 15–16, 867–871.

[1] playing a role for the Boltzmann equation similar to that of Leray's solutions for the 3D incompressible Navier-Stokes equations.