

COMPLEX DIFFERENTIAL EQUATIONS – Example Sheet 3

TKC Lent 2008

1. Show that each of the following equations has a fixed singularity, where, along a suitable path approaching the singularity, the solutions have no limits.

$$f'(z) = z^{-2}f(z)$$

$$f'(z) = i(1 - z)^{-1}f(z)$$

$$f'(z) = f(z) .$$

2. Give an example of a singular point of a differential equation where there is at least one solution that is analytic at that point.
3. Find all of the fixed singularities of

$$(z + f(z))f'(z) - z + f(z) = 0$$

and determine the character of the solutions near these points. Show that there are movable branch points of order 1.

4. Find the fixed singular points of

$$f'(z) = P(z, f(z))$$

where P is a polynomial in 2 variables.

5. Find the singularities of

$$f'(z) = z^{1/2} + z^{3/2}f(z) - f(z)^2 .$$

6. Show that

$$f'(z) = z^3 + f(z)^3 ; \quad f(0) = w_0$$

has movable branch points and find their order. If $w_0 > 0$, the branch point $b(w_0)$ nearest to the origin lies on the positive real axis. How does $b(w_0)$ change as w_0 increases? Where are the fixed singular points of the differential equation, if any?